ROUND I: Similarity and Pythagorean relations

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Triangle ABC is a 30-60-90 triangle whose shortest side is 10 inches long. A similar triangle, FTP, has an area nine times as big as ABC. What is the length of the hypotenuse of triangle FTP?

2. A diagonal of a rectangle is 5 inches longer than 4 times the width of the rectangle. Find the length of the rectangle if it is 1 inch less than the length of the diagonal.

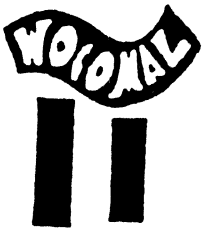
3. A sphere is inscribed in a right circular cone with base radius 5 and height 12. Find the radius of the sphere.

ANSWERS

1. (1 pt) _____ in.

2. (2 pts) _____ in

3. (3 pts) _____



ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. A maple tree is less than 500 years old. An elm tree is 800 years older than the maple tree. A redwood tree is 1200 years older than the elm tree The sum of the ages of the three trees is 3700 years. How old is the maple tree?

2. Solve for x: $10x = \dfrac{(x+2)(x^2-9)}{x^2-x-6}$.

3. Several ordered pairs of positive integers (x,y) satisfy $x^2 - y^2 = 45$. Find the pair with the greatest sum.

ANSWERS

1. (1 pt) _____ years

2. (2 pts) $x =$ _____

3. (3 pts) (,) _____

Auburn, Leicester, West Boylston

ROUND III: Functions

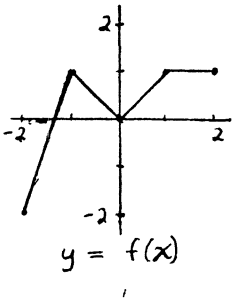
NO CALCULATOR USE

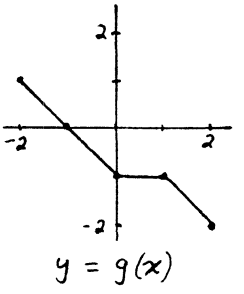
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If $Q(x) = x^2 + 3x + 1$ and $P(x) = .0732$, find $P(Q(x))$.

2. If $g(x) = 1 - x^2$ and $f(g(x)) = \frac{1 - x^2}{x^2}$ when $x \neq 0$, find the value of $f(\frac{1}{2})$.

3. The graphs of f and g are from linear segments joining lattice points. Draw the graph of $y = f(g(x))$.



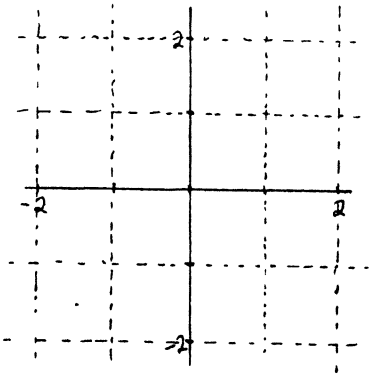


ANSWERS

1. (1 pt) _____

2. (2 pts) _____

3. (3 pts)



Mass. Academy, South, Worcester Academy

ROUND IV: Combinatorics

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. In how many ways can you choose 5 books from a set of 14 distinct books? Order does not matter.

2. Duncan wants to buy 4 donuts from an ample supply of 3 types of donuts: glazed, chocolate, and powdered. How many different selections are possible? Order does not matter.

3. How many positive integers less than or equal to 4000 can be written down without using the digits 7, 8, or 9?

ANSWERS

1. (1 pt) _____

2. (2 pts) _____

3. (3 pts) _____

Algonquin, Burncoat, Northbridge

February 6, 2002 WOCOMAL Varsity Meet

ROUND V: Analytic geometry

NO CALCULATOR USE

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Find the trisection point of the segment joining A(-6, 4) and B(12, -8) which is closer to B.

2. An ellipse has center at (3,4), a focus at (6,4), and a vertex at (10, 4). Write its equation in the form $\frac{(x-h)^2}{a^2 or b^2} + \frac{(y-k)^2}{b^2 or a^2} = 1$

3. Find the point(s) of intersection of the circle with center at (6, 10) and radius 4 with the asymptotes of the hyperbola with equation $x^2 - y^2 = 9$.

ANSWERS

1. (1 pt) _____(,)_____

2. (2 pts) _____

3. (3 pts) _____

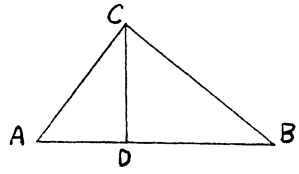
Auburn, Quaboag, Shrewsbury

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ROUND ANSWER SHEET

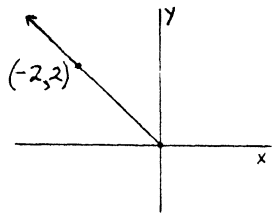
2 points each

1. If $m\angle ACB = 90$, $\overline{CD} \perp \overline{AB}$, $AD = 4$, and $AC = 4\sqrt{17}$, find the area of ΔABC.



2. Determine the value of $v - z$ if:
$$v + w = 20$$
$$w + x = 18$$
$$x + y = 16$$
$$y + z = 14$$

3. If the ray shown is the graph of $f(x)$, sketch the graph of $f^{-1}(x)+2$.



4. There are exactly 3 ways to add 4 odd numbers to get 10 Assume that re-arranging the order of the numbers does not give a new solution. How many ways are there of adding 8 odd numbers to get 20?

5. In an x-y coordinate system, the line through A(1, 4) and B(-5,0) is given a new one-dimensional coordinate system with A having coordinate 0 and B having coordinate 1. What are the coordinates in the x-y system of the point in the new system with coordinate 2.5?

6. If x is an integer, graph the solution set of the inequality $(4-|x|)^2 \leq 3$.

7. A rectangle has length L and width W. Find, in terms of L, W, and D, the increase in length required to keep the area unchanged when its width is decreased by D.

8. In a bag there is a regular triangle, a regular quadrilateral, a regular hexagon, and a regular octagon. The bag is opened and one angle is selected at random from the over 20 equally likely angles. What is the probability that the measure of that angle is a multiple of 45? Give your answer as a reduced fraction.

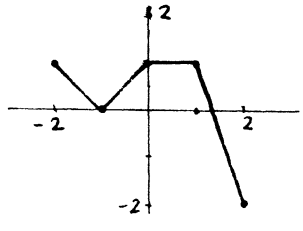
9. House numbering digits may be purchased at the rate of 5 digits for $1.00 or 25 cents each. Pretend that you live on a very long street in a house numbered $(5^{73})(2^{80})$. What is the cheapest, in dollars and cents, for which your number when written out without exponents can be purchased?

Algonquin, Auburn, Bancroft, Bromfield, Hudson, Quaboag, Shrewsbury, Worcester Academy, QSC

ROUND I
1. 1 pt 60 in
sim Pythag
2. 2 pts 40 in
3. 2 pts $\frac{10}{3}$ or $3\frac{1}{3}$ or $3.\overline{3}$

ROUND II
1. 1 pt 300 years
alg 1
2. 2 pts $x = \frac{1}{3}$ or $.\overline{3}$
3. 2 pts (23, 22)

ROUND III
1. 1 pt .0732
f(x)
2. 2 pts 1



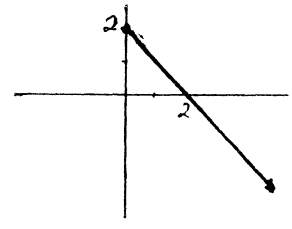
3. 2 pts

ROUND IV
1. 1 pt 2002
comb
2. 2 pts 15
3. 2 pts 1372

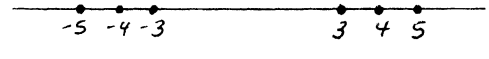
ROUND V
1. 1 pt (6, -4)
analy
2. 2 pts $\frac{(x-3)^2}{49} + \frac{(y-4)^2}{40} = 1$
3. 2 pts (6, 6) and (12, 12)

TEAM ROUND 2 pts each

1. 544
2. 4



3.
4. 11
5. (-14, -6)
6.



7. $\frac{LD}{W-D}$ or equivalent
8. $\frac{4}{7}$
9. $15.25

February 6, 2002 WOCOMAL Varsity Meet BRIEF SOLUTIONS

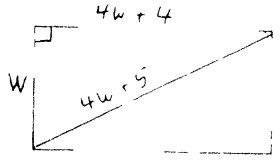
ROUND I

1 Hypoten. of $\triangle ABC = 2 \cdot 10 = 20$

Area ratio $= \frac{9}{1} = (\text{side ratio})^2$

Side ratio $\frac{3}{1}$ of hypot of $\triangle FTP$ $3 \cdot 20 = 60$ in

2

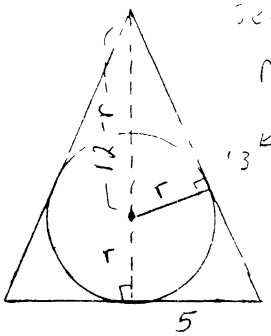


$w^2 + (4w+4)^2 = (4w+5)^2$

$w^2 - 8w - 9 = (w-9)(w+1) = 0$

$w = 9$ our length $= 4 \cdot 9 + 4 = 40$ in

3



Section along axis of cone

Pyth thm

$\sim \triangle$s gets

$\frac{12-r}{13} = \frac{r}{5}$

$60 - 5r = 13r$

$\frac{60}{18} = r = \frac{10}{3}$

ROUND II

1 m = maple tree age

$e = m + 80$

$r = e + 1200 = m + 2000$

$3700 = 3m + 2200 \Rightarrow m = 300$ yrs

2. $10x = \frac{(x+2)(x^2-9)}{x^2-x-6} = \frac{(x+2)(x+3)(x-3)}{(x+2)(x-3)}$

$10x = x + 3 \Rightarrow x = \frac{1}{3}$

3. $(x+y)(x-y) = 45$ Want biggest $x+y$

$9 \cdot 5$

$15 \cdot 3$ → Take $x+y = 45$

$45 \cdot 1$ $x-y = 1$

$2x = 46$

$x = 23$

$y = 22$

Pair is $(23, 22)$

ROUND III

1 $x^2 + 5x + 1$ is irrelevant

$f(\text{any number}) = 0.732$

2 Since we want $f(\frac{1}{2})$, set $g(x) = \frac{1}{2}$.

$1 - x^2 = \frac{1}{2}$ $\quad x^2 = \frac{1}{2}$

Then using $f(g(x)) = \frac{1-x^2}{x^2}$ gets

$f(\frac{1}{2}) = \frac{1 - \frac{1}{2}}{\frac{1}{2}} = 1$

3.

x	$g(x)$	$f(g(x))$
-2	1	1
-1	0	0
0	-1	1
1	-1	1
2	-2	-2

↑ From g graph ↑ From f graph

Use columns 1 and 3 for answer points

Connect dots by line segments after thinking about in between x-values

ROUND IV

1. Straightforward combinations situation, but with a nice answer

$14C5 = \binom{14}{5} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2002$

2. $g + c + p = 4$

(g, c, p) triples

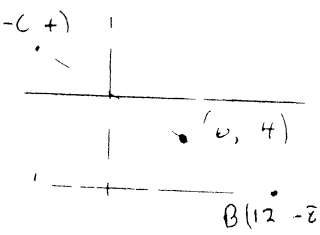
4,0,0	1,3,0	2,2,0
0,4,0	1,0,3	2,0,2
0,0,4	0,3,1	0,2,2
3,0,1	0,1,3	2,1,1
3,1,0		1,2,1
		1,1,2

15 possible

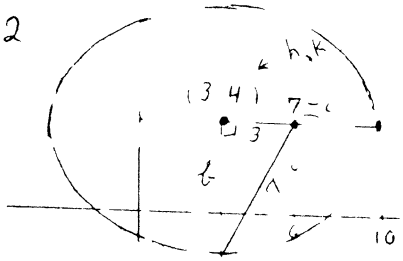
ROUND IV Cont

3 The numbers of possible digits in each position are multiplied $4\ 7\ 7\ 7 = 1372$
We subtract 1 because this counts 0, but we add 1 to include 4000

ROUND V

1 

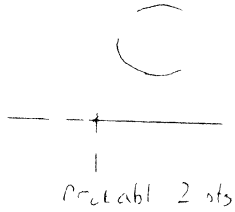
x dist = 18
Subtract $\frac{1}{3}$ of 18, 6 to 6 from 12 to get 6
y dist = 12
Add $\frac{1}{3}$ of 12, 4 to -6 to get -4

2 

$3^2 + b^2 = 7^2 (= c^2)$
gets $b^2 = 40$
longer axis is horizontal

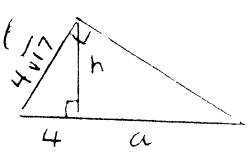
$$\frac{(x-3)^2}{49} + \frac{(y-4)^2}{40} = 1$$

3 Circle $(x-6)^2 + (y-10)^2 = 16$
Hyperbola asymptotes from $x^2 - y^2 = 0$
$y = \pm x$
Picture shows possible intersections from graph
$(x-6)^2 + (x-10)^2 = 16$

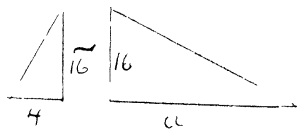


$x^2 - 16x + 60 = 0$
$(x-6)(x-10) = 0$
$x = 6$ or $x = 10$
Intersection pts are (6, 6) and (10, 10)

TEAM ROUND

1 

$h^2 + 16$ 16 17 gets $h = 16$



Similar Δ, gets $\frac{4}{16}$ $\frac{16}{a}$ and $a = 64$

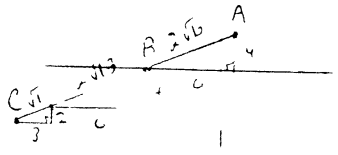
Area of $\triangle ABC = \frac{1}{2}\ 66\ 16 = 544$

TEAM ROUND cont

2
$$\begin{aligned} V + W &= 20 \\ W + X &= 18 \\ V - X &= 2 \\ X + Y &= 16 \\ V - Y &= 18 \\ Y + Z &= 14 \\ V - Z &= 4 \end{aligned}$$

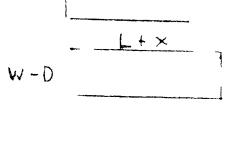
3 For $f^{-1}(x)$, reflect the f graph across the line $y = x$ Then for +2, move the resulting ray up 2 units

4 Make and find eleven ways
13 ones 7 1s
3 11s, 6 1s
2, 4 6 1s
7, 7 6 1s
3 3 9 5 1s
3 5 7 5 1s
5 5 5 5 1s
3 3 3, 7, 4 1s
3 3 5 5 4 1s
3 3 3 3 5, 3 1s
6 3s and 2 1s

5 

To keep same slope and make 1 5 times to find pt C by going 4 left and 6 down from B(-5,0) getting to (-14,-6)

6 $-\sqrt{3} < 4 - |x| < \sqrt{3}$
$-4 - \sqrt{3} < -|x| < 4 + \sqrt{3}$
$4 + \sqrt{3} > |x| > 4 - \sqrt{3}$ or approx $5.7 > |x| > 2.3$
Integer values are $\pm 3, \pm 4, \pm 5$

7 

$LW = (L + x)(W - D)$
$LW = LW + xW - LD$ $\times D$
$LD = x(W - D)$
$x = \frac{LD}{W - D}$

8 3 60s, 4 90s 6 120s, and 8 135s are available 12 of the 21 are multiples of 45 $\frac{12}{21} = \frac{4}{7}$

9 $5^{73} 2^{80} = 5^{73} 2^{73} 2^7 = 10^{73} \cdot 2^7$
This 128×10^{73} 1.28×10^{75} which has 76 digits $76 - 5 = 15\frac{1}{5}$ You can buy 75 digits for \$15 and the extra one for 25¢ \$15.25